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ACOUSTIC RADIATION FROM A DOUBLE HULL STRUCTURE

by

Martin L. Pollack and Jerome M. Klosner

POLYTECHNIC INSTITUTE OF BROOKLYN

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The fluid is assumed to be inviscid and compressible and its motion is governed by the wave equation; the shell response is determined by using classical shell theory. Coupling of these equations arise from two interface conditions, namely: (1) the fluid pressure is exerted on the shell, and, (2) the radial fluid and shell velocities are equal.

The results indicate that in certain regions of the frequency ratio ($\Omega \frac{L}{\pi}$), the pressure in the outer fluid at the interface was somewhat greater for the double shell than for the single shell. However, the infinite response at the nonradiating frequency ($\Omega = \Omega_1$) no longer was present for the double shell. Furthermore, the introduction of structural damping did result in improved transmission loss properties for the double shell.

Finally, it should be pointed out that greatly improved transmission loss properties can be expected for the double shell construction by choosing the appropriate combination of materials for the inner and outer shells. Such a parametric study will be forthcoming in the future.

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Abstract

Acoustic radiation characteristics are determined for a double shell structure, consisting of two circular cylindrical shells separated by, and immersed in, an acoustic medium.

The fluid is assumed to be inviscid and compressible and its motion is governed by the wave equation; the shell response is determined by using classical shell theory. Coupling of these equations arise from two interface conditions, namely: (1) the fluid pressure is exerted on the shell, and, (2) the radial fluid and shell velocities are equal.

The results indicate that in certain regions of the frequency ratio ($\Omega \frac{L}{c}$), the pressure in the outer fluid at the interface was somewhat greater for the double shell than for the single shell. However, the infinite response at the nonradiating frequency ($\Omega = \Omega_1$) no longer was present for the double shell. Furthermore, the introduction of structural damping did result in improved transmission loss properties for the double shell.

Finally, it should be pointed out that greatly improved transmission loss properties can be expected for the double shell construction by choosing the appropriate combination of materials for the inner and outer shells. Such a parametric study will be forthcoming in the future.

List of Symbols

u,v,w = nondimensional displacements in the axial, circumferential, and radial directions respectively.

x,θ,r = nondimensional axial, circumferential, and radial coordinates respectively.

Φ = fluid velocity potential.

V_r = radial acoustic velocity.

p_r = radial acoustic pressure.

ρ,ρ_s = mass density of the fluid and shell respectively.

c,c_s = velocity of sound in the fluid and shell respectively.

a,b = radius of the outer and inner shell respectively.

P_i = amplitude of the internal pressure: $p_i = P_i \cos n\theta \cos \frac{\pi x}{L} e^{i\Omega\tau}$

p = the total external pressure acting on the shell.

$H_n^{(1)}, H_n^{(2)}$ = Hankel functions of nth order of the first and second kinds respectively.

I_n,K_n = modified Bessel functions.

Ω = angular frequency

E_s = elastic modulus of shell.

n = circumferential wave number.

L = nondimensional half-wavelength.

U,V,W = nondimensional amplitudes of the corresponding displacements.

τ = nondimensional time (τ=ct/a)

h = thickness of the shell.

w_{static} = nondimensional static radial displacement of the shell.

W_{static} = nondimensional amplitude of the static radial displacement of the shell.

ν = Poisson's ratio.

g = coefficient of structural damping.

$(\ )_{i,o}$ = refers to the inner or outer shell and fluid respectively.

INTRODUCTION

The Navy has long been interested in obtaining accurate information on the transmission of sound through water. Such knowledge is useful in designing sonar gear for echo ranging, as well as in providing data which is helpful in determining the choice of submarine tactics.

During World War II a large amount of information was obtained on the propagation of underwater sound[1]. Since that period, the research effort has continued unabated, and has been instrumental in developing a more silent submarine, that is, one for which the transmission loss of the vehicle's noise is large. Typical noise reduction techniques include dynamic absorbers and isolators to reduce machinery induced vibrations, streamlining the hull so as to minimize flow noises, corrective propulsion designs to reduce cavitation and prevent "singing" propellers, acoustic filters for pumps, and structural damping treatment to reduce hull vibrations.

In particular, during the last decade or two, a series of theoretical investigations[2,3,4,5,6] have been carried out in which the radiation due to hull vibration has been analyzed. The present study is a continuation of this effort to obtain a basic understanding of the radiation characteristics of structures. Of prime interest is the transmission loss characteristics of a double hull construction, including the effect of structural damping.

Theory

PART I. SINGLE SHELL

A. Basic Equations

Consider the response of a circular cylindrical shell acted upon by an internal pressure, $p_i = P_i \cos n\theta \cos \frac{\pi x}{L} e^{i\Omega\tau}$ and submerged in a fluid which is assumed to be compressible and inviscid. (see Fig. 1)

The motion of the shell is described by the following nondimensional Donnell shell equations.[7]

$$\frac{h^2}{12a^2} \nabla^4 w + \left(w - \frac{\partial v}{\partial \theta} - \frac{\nu \partial u}{\partial x}\right) - \frac{a(1-\nu^2)}{hE_s} p + (1-\nu^2)\left(\frac{c}{c_s}\right)^2 \frac{\partial^2 w}{\partial \tau^2} = 0 \qquad (1a)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{(1-\nu)}{2} \frac{\partial^2 u}{\partial \theta^2} + \frac{(1+\nu)}{2} \frac{\partial^2 v}{\partial x \partial \theta} - \frac{\nu \partial w}{\partial x} + \frac{a(1-\nu^2)}{hE_s} X - (1-\nu^2)\left(\frac{c}{c_s}\right)^2 \frac{\partial^2 u}{\partial \tau^2} = 0 \qquad (1b)$$

$$\frac{\partial^2 v}{\partial \theta^2} + \frac{(1-\nu)}{2} \frac{\partial^2 v}{\partial x^2} + \frac{(1+\nu)}{2} \frac{\partial^2 u}{\partial x \partial \theta} - \frac{\partial w}{\partial \theta} + \frac{a(1-\nu^2)\Theta}{hE_s} - (1-\nu^2)\left(\frac{c}{c_s}\right)^2 \frac{\partial^2 v}{\partial \tau^2} = 0 \qquad (1c)$$

where u, v, w are the nondimensional components of the displacement, x is the nondimensional axial coordinate, $\tau \left(= \frac{ct}{a}\right)$ is the nondimensional time, and c and c_s is the velocity of sound in the fluid and shell respectively. p, X, Θ are the external forces acting on the shell per unit area in the radial, axial, and circumferential directions, respectively. All length dimensions are nondimensionalized with respect

to the shell radius a.

The governing fluid field equation can be expressed as

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial \tau^2} \tag{2}$$

and the corresponding radiated radial velocity and pressure are

$$V_r = -\frac{1}{a}\frac{\partial \varphi}{\partial r} \tag{3}$$

and

$$p_r = \frac{\rho c}{a}\frac{\partial \varphi}{\partial \tau} \tag{4}$$

Thus the total pressure acting on the shell surface can be written as

$$p = -p_i + p_r \tag{5}$$

which is the sum of the internal and radiated pressures.

Since the fluid velocity and the shell velocity are equal at the interface,

$$\frac{\partial w}{\partial \tau} = -\frac{1}{c} V_r = \frac{1}{ac}\frac{\partial \varphi}{\partial r}\Big|_{r=1} \tag{6}$$

B. Solution

Solutions are assumed of the form

$$\begin{aligned} u &= U \cos n\theta \sin \frac{\pi x}{L} e^{i\Omega\tau} \\ v &= V \sin n\theta \cos \frac{\pi x}{L} e^{i\Omega\tau} \\ w &= W \cos n\theta \cos \frac{\pi x}{L} e^{i\Omega\tau} \\ \varphi &= R(r) \cos n\theta \cos \frac{\pi x}{L} e^{i\Omega\tau} \end{aligned} \tag{7}$$

From Eq. (2) one obtains the following differential equation:

$$\frac{d^2R(r)}{dr^2} + \frac{1}{r}\frac{dR(r)}{dr} + (k^2 - \frac{n^2}{r^2})\, R(r) = 0 \qquad (8)$$

where $\qquad k = \sqrt{\Omega^2 - (\frac{m\pi}{L})^2} \qquad (9)$

The solution of Eq. (8) which satisfies the radiation condition[4] can be expressed as

$$R(r) = A_n\, F\,(r) \qquad (10)$$

where

$$F(r) = \begin{cases} H_n^{(2)}(kr) & , \text{ if } \Omega > \frac{m\pi}{L} \\ r^{-n} & , \text{ if } \Omega = \frac{m\pi}{L} \;, \quad n \neq 0, 1^{*} \\ K_n(\kappa r) & , \text{ if } \Omega < \frac{m\pi}{L} \end{cases} \qquad (11)$$

and $\qquad \kappa = \sqrt{(\frac{m\pi}{L})^2 - \Omega^2} \qquad (12)$

The interface condition (5) yields

$$A_n = \frac{a c i \Omega W}{F'(1)} \qquad (13)$$

where $\qquad F'(1) = \frac{dF}{dr}\Big|_{r=1}$

Substituting Eqs. (5) and (7) into Eq. (1), and letting $X = \Theta = 0$, then results in the following displacement expressions

$$U = \frac{a(1-\nu^2)\,P_i}{hE_s}\left\{\frac{[illegible]}{(\Omega^2-\omega_1^2)(\Omega^2-\omega_2^2)(\Omega^2-\omega_3^2) + K\Xi}\right\} \qquad (14)$$

$$V = \frac{a(1-\nu^2)\,P_i}{hE_s}\left\{\frac{[illegible]}{(\Omega^2-\omega_1^2)(\Omega^2-\omega_2^2)(\Omega^2-\omega_3^2) + K\Xi}\right\} \qquad (15)$$

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*The solution for n = 0 does not satisfy the radiation condition. Also the total kinetic energy in the medium for n = 0,1 for these cases is not finite (see Ref. 4).

$$W = -\frac{a(1-\nu^2)P_i}{hE_s}\left\{\frac{\xi}{(\Omega^2-\omega_1{}^2)(\Omega^2-\omega_2{}^2)(\Omega^2-\omega_3{}^2)+K\xi}\right\} \tag{16}$$

where
$$\alpha = \left(\frac{1+3\nu}{2}\right) n^2 \frac{\pi}{L} + \frac{\nu(1-\nu)}{2}\left(\frac{\pi}{L}\right)^3 - \nu\frac{\pi}{L}\left(\frac{c}{c_s}\right)^2(1-\nu^2)\Omega^2 \tag{17}$$

$$\beta = -\left(\frac{\nu^2+\nu+2}{2}\right) n \left(\frac{\pi}{L}\right)^2 - \frac{(1-\nu)n^3}{2} + n\left(\frac{c}{c_s}\right)^2(1-\nu^2)\Omega^2 \tag{18}$$

$$\xi = \frac{(1-\nu)}{2}\left[\left(\frac{\pi}{L}\right)^2 + n^2\right]^2 + \left(\frac{c}{c_s}\right)^2(1-\nu^2)\Omega^2\left[n^2+\left(\frac{\pi}{L}\right)^2\right]\frac{(\nu-3)}{2} + \left[\left(\frac{c}{c_s}\right)^2(1-\nu^2)\Omega^2\right]^2 \tag{19}$$

$$K = \begin{cases} \dfrac{(1-\nu^2)}{k}\left(\dfrac{c}{c_s}\right)^2\left(\dfrac{a}{h}\right)\dfrac{\Omega^2 H_n^{(2)}(k)\quad(\rho/\rho_s)}{\left[H_{n-1}^{(2)}(k) - \frac{n}{k}H_n^{(2)}(k)\right]}, & \text{for } \Omega > \dfrac{\pi}{L} \\[2ex] -\dfrac{(1-\nu^2)}{n}\left(\dfrac{c}{c_s}\right)^2\left(\dfrac{a}{h}\right)\Omega^2\quad(\rho/\rho_s), & \text{for } \Omega = \dfrac{\pi}{L} \\[2ex] \dfrac{(1-\nu^2)}{\kappa}\left(\dfrac{c}{c_s}\right)^2\left(\dfrac{a}{h}\right)\dfrac{\Omega^2 K_n(\kappa)\quad(\rho/\rho_s)}{\left[-\frac{n}{\kappa}K_n(\kappa) - K_{n-1}(\kappa)\right]}, & \text{for } \Omega < \dfrac{\pi}{L} \end{cases} \tag{20}$$

and $\omega_k (k = 1,2,3)$ are the three in vacuo frequencies for a given length L and number n.

The amplitude of the static deflection, W_{static}, obtained by letting $\Omega \to 0$, is

$$W_{static} = \frac{a(1-\nu^2)P_i}{hE_s}\left\{\frac{\frac{(1-\nu)}{2}\left[\left(\frac{\pi}{L}\right)^2+n^2\right]^2}{(\omega_1\,\omega_2\,\omega_3)^2}\right\} \tag{21}$$

Finally, from Eq.(4), the radiated pressure is

$$p_r = \left(\frac{\rho}{\rho_s}\right)\left(\frac{c}{c_s}\right)^2\Omega^2(1-\nu^2)\left(\frac{a}{h}\right)\frac{F(r)}{F'(1)}\times P_i$$
$$\times\left\{\frac{\xi}{(\Omega^2-\omega_1^2)(\Omega^2-\omega_2^2)(\Omega^2-\omega_3^2)+K\xi}\right\}\cos n\theta\,\cos\frac{\pi x}{L}\,e^{i\Omega\tau} \tag{22}$$

C. Limiting Case : Pure Breathing Mode ($n = 0, L \to \infty$)

Considering that the radial displacement is independent of the spatial coordinates, and neglecting the axial inertia term, Eqs. (1b) and (1c) result in

$$\frac{\partial u}{\partial x} = \nu w \tag{23}$$

and

$$v = 0 \tag{24}$$

while equation (1a) yields

$$\left(\frac{c}{c_s}\right)^2 \frac{d^2 w}{d\tau^2} + w = \frac{a}{hE_s}\, p \tag{25}$$

For this case then

$$\frac{w}{w_{static}} = \frac{e^{i\Omega\tau}}{\left[1 - \left(\Omega \frac{c}{c_s}\right)^2 - \left\{\frac{\rho}{\rho_s}\left(\frac{c}{c_s}\right)^2 \Omega \frac{H_0^{(2)}(\Omega)}{H_1^{(2)}(\Omega)} \frac{a}{h}\right\}\right]} \tag{26}$$

where

$$w_{static} = -\frac{a p_i}{hE_s} \tag{27}$$

PART II DOUBLE SHELL

A. Basic Equations

Consider two concentric circular cylindrical shells separated by, and, immersed in an acoustic medium. An internal pressure, $p_i = P_i \cos n\theta \cos \frac{\pi x}{L} e^{i\omega\tau}$ acts on the inner shell as illustrated in Fig. 2 .

Since all length dimensions are nondimensionalized with respect to the outer shell radius a, the Donnell equations describing the motion of the inner shell of radius b are

$$\frac{h_i^2}{12a^2}\left(\frac{b}{a}\right)\nabla^4 w_i + \left[\left(\frac{a}{b}\right) w_i - \left(\frac{a}{b}\right)\frac{\partial v_i}{\partial \theta} - \nu_i \frac{\partial u_i}{\partial x}\right] = \frac{b(1-\nu_i^2)p}{h_i E_{s_i}} - (1-\nu_i^2)\left(\frac{c}{c_{s_i}}\right)^2 \frac{b}{a}\frac{\partial^2 w_i}{\partial \tau^2} \tag{28a}$$

$$\frac{\partial^2 u_i}{\partial x^2} + \frac{(1-\nu_i)}{2}\left(\frac{a}{b}\right)^2\frac{\partial^2 u_i}{\partial \theta^2} + \frac{(1+\nu_i)}{2}\left(\frac{a}{b}\right)\frac{\partial^2 v_i}{\partial x\partial\theta} - \frac{a}{b}\nu_i\frac{\partial w_i}{\partial x} = (1-\nu_i^2)\left(\frac{c}{c_{s_i}}\right)^2\frac{\partial^2 u_i}{\partial \tau^2} - \frac{a(1-\nu_i^2)}{h_i E_{s_i}} X \tag{28b}$$

$$\left(\frac{a}{b}\right)^2\frac{\partial^2 v_i}{\partial \theta^2} + \frac{(1-\nu_i)}{2}\frac{\partial^2 v_i}{\partial x^2} + \frac{(1+\nu_i)}{2}\frac{a}{b}\frac{\partial^2 u_i}{\partial x\partial\theta} - \left(\frac{a}{b}\right)^2\frac{\partial w_i}{\partial \theta} = (1-\nu_i^2)\left(\frac{c}{c_{s_i}}\right)^2\frac{\partial^2 v_i}{\partial \tau^2} - \left(\frac{a}{h_i}\right)\frac{(1-\nu_i^2)\,\Theta}{E_{s_i}} \tag{28c}$$

The interface condition at the inner shell is expressed as,

$$\frac{\partial w_i}{\partial \tau} = \frac{1}{ac}\frac{\partial \varphi_i}{\partial r}\Big|_{r=b/a} \tag{29}$$

while at the surface of the outer shell the radial velocities of the separating fluid, the outer surrounding fluid, and the shell are all equal. Thus

$$\frac{\partial w_o}{\partial \tau} = \frac{1}{ac}\frac{\partial \phi_o}{\partial r}\Big|_{r=1} = \frac{1}{ac}\frac{\partial \phi_i}{\partial r}\Big|_{r=1} \tag{30}$$

where the subscripts o and i refer to the outer and inner shells, and to the surrounding and separating fluids respectively.

B. Solution

Solutions are again assumed of the form of Eqs.(7) for both the inner and outer shells and fluids.

The solution of Eq.(8) then yields for the outer fluid

$$R_o(r) = B_n\ F(r) \tag{31}$$

where

$$F(r) = \begin{cases} H_n^{(2)}(kr) & , \text{ if } \Omega > \frac{\pi}{L} \\ r^{-n} & , \text{ if } \Omega = \frac{\pi}{L}^{*} \\ K_n(\kappa r) & , \text{ if } \Omega < \frac{\pi}{L} \end{cases} \tag{32}$$

and for the inner fluid

$$R_i(r) = C_n G(r) + D_n\ F(r) \tag{33}$$

where

$$G(r) = \begin{cases} H_n^{(1)}(kr) & , \text{ if } \Omega > \frac{\pi}{L} \\ r^{n} & , \text{ if } \Omega = \frac{\pi}{L} \\ I_n(\kappa r) & , \text{ if } \Omega < \frac{\pi}{L} \end{cases} \tag{34}$$

*For the outer fluid, no steady-state solution for $\Omega = \frac{\pi}{L}$ and $n = 0,1$ exists (see footnote pg.4).

From interface conditions (29) and (30)

$$B_n = \frac{aci\Omega W_o}{F'(1)} \tag{35}$$

$$C_n = \frac{aci\Omega[W_i - W_o \frac{F'(b/a)}{F'(1)}]\, F'(1)}{G'(b/a)F'(1) - G'(1)F'(b/a)} \tag{36}$$

and

$$D_n = \frac{i\Omega W_o ac}{F'(1)} - \frac{aci\Omega[W_i - W_o \frac{F'(b/a)}{F'(1)}]\, G'(1)}{G'(b/a)\,F'(1) - G'(1)\,F'(b/a)} \tag{37}$$

Substituting Eqs. (5) and (7) into Eqs. (1) and (28) and letting $x = \theta = 0$, then results in the following displacements for the inner and outer shells

$$W_i = -\frac{[a(1-\nu_i^2)P_i + b(1-\nu_i^2)\rho c^2[\frac{\Omega^2}{F'(1)}](F(b/a) - F'(b/a)\Delta_2)W_o\Xi]}{h_i E_{s_i}[(\Omega^2-\omega_{i_1}^2)(\Omega^2-\omega_{i_2}^2)(\Omega^2-\omega_{i_3}^2) + \Xi(\frac{b}{a})(\frac{a}{h_i})(1-\nu_i^2)(\frac{c}{c_{s_i}})^2(\frac{\rho}{\rho_{s_i}})]} \tag{38}$$

$$W_o = -\frac{(\rho/\rho_{s_o})(c/c_{s_o})^2(a/h_o)(1-\nu_o^2)\Delta_1\bar{\Xi}W_i}{(\Omega^2-\omega_{o_1}^2)(\Omega^2-\omega_{o_2}^2)(\Omega^2-\omega_{o_3}^2) - (\frac{\rho}{\rho_{s_o}})(\frac{c}{c_{s_o}})^2(\frac{a}{h_o})^2(1-\nu_o^2)\frac{F'(b/a)}{F'(1)}\Delta_1\bar{\Xi}} \tag{39}$$

where $\bar{\Xi}$ is obtained from Ξ by replacing n by $\frac{a}{b}n$ in Eq.(19), and

$$\Delta_1 = \frac{G'(1)\,F(1) - F'(1)\,G(1)}{G'(b/a)\,F'(1) - G'(1)\,F'(b/a)} \tag{40}$$

$$\Delta_2 = \frac{F'(1)G(b/a) - G'(1)\,F(b/a)}{G'(b/a)\,F'(1) - G'(1)\,F'(b/a)} \tag{41}$$

One could find W_o and W_i by solving Eqs. (38) and (39) simultaneously, and substituting Eqs. (32) and (34)

Again, for convenience it is desirable to nondimensionalize the shell displacement with respect to the static radial displacement of the shell. In order to determine this displacement, it is required to know the change in the pressure of the enclosed fluid, which, of course, is a function of the change of the volume contained between the two shells. This volume change may be expressed as:

$$\Delta V = \int_0^{2L}\int_0^{2\pi} w_o a^2 \, d\theta \, dx - \int_0^{2L}\int_0^{2\pi} w_i b^2 \, d\theta \, dx$$

However, the harmonic form of the displacement functions results in the vanishing of the above integrals except for the limiting case $n = o$, $L \to \infty$, which describes the plane breathing mode. Thus in the static case the fluid pressure of both the separating and surrounding fluids vanishes, and therefore letting $\Omega = 0$ in Eqs. (38) and (39) results in

$$W_{o_{static}} = 0 \qquad (42)$$

$$W_{i_{static}} = + \frac{b(1-\nu_i^2)}{h_i E_{s_i}} P_i \left\{ \frac{(1-\nu_i)}{2} \left[\left(\frac{\pi}{L}\right)^2 + \left(n \frac{a}{b}\right)^2 \right]^2 / (\omega_{i_1} \omega_{i_2} \omega_{i_3})^2 \right\} \qquad (43)$$

Finally, the inner and outer fluid pressures are, from Eq. 4,

$$p_{ri} = - \rho c^2 \Omega^2 \left\{ \frac{W_i [F'(1)G(r) - G'(1)F(r)] + W_o \left[\frac{F'(\frac{b}{a})}{F'(1)}\right] [G'(1)F(r) - F'(1)G(r)]}{G'(b/a)F'(1) - G'(1)F'(b/a)} + \right.$$

$$\left. + \frac{F(r)}{F'(1)} \right\} \cos n\theta \cos \frac{\pi x}{L} e^{i\Omega\tau} \qquad (44)$$

and

$$p_{ro} = -\rho c^2 \Omega^2 \frac{W_o F(r)}{F'(1)} \cos n\theta \cos \frac{\pi x}{L} e^{i\Omega\tau} \tag{45}$$

C. Limiting Case: Pure Breathing Mode ($n = 0$, $L \to \infty$)

The axisymmetric, axially independent equation of motion yields for the outer shell

$$\left(\frac{c}{c_{s_o}}\right)^2 \frac{d^2 w_o}{d\tau^2} + w_o = \frac{a}{h_o E_{s_o}} \left[\frac{\rho c}{a} \frac{\partial \varphi_o}{\partial \tau}(r = 1) - \frac{\rho c}{a} \frac{\partial \varphi_i}{\partial \tau}(r = 1)\right] \tag{46}$$

and for the inner shell

$$\left(\frac{b}{a}\right)^2 \left(\frac{c}{c_{s_i}}\right)^2 \frac{d^2 w_i}{d\tau^2} + w_i = \frac{b^2}{a h_i E_{s_i}} \left[-p_i e^{i\Omega\tau} + \frac{\rho c}{a} \frac{\partial \varphi_i}{\partial \tau}(r = b/a)\right] \tag{47}$$

The solution of Eqs.(46) and (47) is

$$W_i = -\frac{\left[b(1-\nu_i^2)\, p_i + b(1-\nu_i^2)\rho c^2 \left[\frac{\Omega}{H_1^{(2)}(\Omega)}\right]\left(H_o^{(2)}(\Omega b/a) - \Omega H_1^{(2)}(\Omega b/a)\delta_2\right) W_o\right]\left[\left(\frac{c}{c_{s_i}}\right)^2 (1-\nu_i^2)\Omega^2\right]^2}{h_i E_{s_i}\left[(\Omega^2-\omega_i^2) + \left(\frac{b}{a}\right)\left(\frac{a}{h}\right)(1-\nu_i^2)\left(\frac{\rho}{\rho_{s_i}}\right)\left(\frac{c}{c_{s_i}}\right)^2 \delta_2 \left(\left(\frac{c}{c_{s_i}}\right)^2 (1-\nu_i^2)\Omega^2\right)^2\right]} \tag{48}$$

and

$$W_o = -\frac{(\rho/\rho_{s_o})(c/c_{s_o})^2 (a/h_o)\Omega^2 (1-\nu_o^2)\delta_1 \left[\left(\frac{c}{c_{s_o}}\right)^2 (1-\nu_o^2)\Omega^2\right] W_i}{(\Omega^2-\omega_o^2) - (\rho/\rho_{s_o})(c/c_{s_o})^2 (a/h_o)\Omega^2 (1-\nu_o^2)\frac{H_1^{(2)}(\Omega b/a)}{H_1^{(2)}(\Omega)}\delta_1\left[\left(\frac{c}{c_{s_o}}\right)^2 (1-\nu_o^2)\Omega^2\right]^2} \tag{49}$$

where

$$\varepsilon_1 = \frac{H_1^{(1)}(\Omega)H_0^{(2)}(\Omega) - H_1^{(2)}(\Omega)H_0^{(1)}(\Omega)}{H_1^{(1)}(\Omega b/a)H_1^{(2)}(\Omega) - H_1^{(1)}(\Omega)H_1^{(2)}(\Omega b/a)}\left(\frac{1}{\Omega}\right) \tag{50}$$

$$\varepsilon_2 = \frac{H_1^{(2)}(\Omega)H_0^{(1)}(\Omega b/a) - H_1^{(1)}(\Omega)H_0^{(2)}(\Omega b/a)}{H_1^{(1)}(\Omega b/a)H_1^{(2)}(\Omega) - H_1^{(1)}(\Omega)H_1^{(2)}(\Omega b/a)}\left(\frac{1}{\Omega}\right) \tag{51}$$

Solving Eqs. (48) and (49) one obtains the amplitudes of the radial displacements of the inner and outer shells, respectively.

For this case the static pressure of the separating fluid is

$$p_{static} = -c^2\rho\,\frac{\Delta V}{V} = -2c^2\rho\left[\frac{(b/a)\,W_{i_{static}} - W_{o_{static}}}{1-(b/a)^2}\right] \tag{52}$$

But equilibrium of the inner and outer shells demands that

$$p_i - p_{static} = E_{s_i}\left(\frac{h_i}{a}\right)\left(\frac{a}{b}\right)^2 W_{i_{static}} \tag{53}$$

and

$$p_{static} = -E_{s_o}\left(\frac{h_o}{a}\right)W_{o_{static}} \tag{54}$$

and thus, it can easily be shown that

$$W_{o_{static}} = \frac{p_i}{E_{s_o}\left(\frac{h_o}{a}\right) + E_{s_i}\left(\frac{h_o}{a}\right)\left(\frac{h_i}{h_o}\right)\left(\frac{a}{b}\right)^2\left\{\frac{1}{2}\left(\frac{c_{s_o}}{c}\right)^2\left(\frac{\rho_{s_o}}{\rho}\right)\left(\frac{h_o}{a}\right)\left(\frac{a}{b}-\frac{b}{a}\right)+\frac{a}{b}\right\}} \tag{55}$$

and

$$W_{i_{static}} = \frac{p_i}{E_{s_i}\left(\frac{h_o}{a}\right)\left(\frac{h_i}{h_o}\right)\left(\frac{a}{b}\right)^2 + \left\{2E_{s_o}\left(\frac{h_o}{a}\right)\left(\frac{b}{a}\right)\Big/\left[\left(\frac{c_{s_o}}{c}\right)^2\left(\frac{\rho_{s_o}}{\rho}\right)\left(\frac{h_o}{a}\right)\left(1-\left(\frac{b}{a}\right)^2\right)+2\right]\right\}} \tag{56}$$

The expressions for the radiated pressures become

$$p_{ri} = -\left\{ \frac{W_i\left[H_1^{(2)}(\Omega)H_o^{(1)}(\Omega r)-H_1^{(1)}(\Omega)H_o^{(2)}(\Omega r)\right] + W_o\left[\frac{H_1^{(2)}(\Omega b/a)}{H_1^{(2)}(\Omega)}\right]\left[H_1^{(1)}(\Omega)H_o^{(2)}(\Omega r)-H_1^{(2)}(\Omega)\,H_o^{(1)}(\Omega r)\right]}{H_1^{(1)}(\Omega b/a)H_1^{(2)}(\Omega) - H_1^{(1)}(\Omega)H_1^{(2)}(\Omega b/a)} + \frac{H_o^{(2)}(\Omega r)}{H_1^{(2)}(\Omega)}\right\} \rho c^2\Omega \cos n\theta \cos\frac{\pi x}{L}\, e^{i\Omega\tau} \tag{57}$$

and

$$p_{ro} = -\rho c^2\,\Omega\,\frac{W_o}{H_1^{(2)}(\Omega)}\cos n\theta\,\cos\frac{\pi x}{L}e^{i\Omega\tau} \tag{58}$$

Structural damping is introduced into the analysis simply by replacing the modulus of elasticity E_s by the complex modulus $\overline{E}_s = E_s(1+ig)$ where g is called the coefficient of structural damping.

Discussion

Calculations were performed at the Polytechnic Institute of Brooklyn on the IBM 360 Digital Computer.

The results for a single steel shell immersed in sea water are shown in Figs. 3, 4, 5, and 6 for a buoyancy factor of two ($\rho a/\rho_s h = 4$), i.e., for a ratio of the mass of the displaced water to the shell mass of 2. These results are identical to those obtained by Bleich and Baron[3] who expressed the responses in terms of the in vacuo modes, and thus their significance will but briefly be discussed. It is observed that a non-radiating free vibrational mode exists for the submerged shell at a frequency $[illegible]_1$ [illegible]/L. For [illegible]/L non-radiating modes are nonexistent, and the fluid acts as a damping mechanism. This is illustrated by the peak responses that occur in the vicinity of the in vacuo shell frequencies, which is typical for a damped system. At frequencies lying between the in vacuo frequencies, the radial shell displacement and fluid pressure vanish, thus indicating that at these frequencies the longitudinal and circumferential modes of the shell act as dynamic dampers. Finally, as was pointed out[3], the displacements and pressures are in phase with the applied force for [illegible] $\frac{[illegible]}{L}$, and therefore no energy is imparted into the fluid, thus resulting in the non-radiating response. When [illegible]/L, the response is radically different: the displacements and pressures are not in phase with the applied force, thereby resulting in energy being radiated into the fluid.

These results have been extended to include the effects of structural damping upon the response. It is seen in Figs. 3, 4, 5, and 6

that the peak responses are significantly reduced as a consequence of the inclusion of the structural damping into the analysis and that these peak responses decrease as the structural damping constant g increases.

Results obtained for the case of the two concentric shells separated by an acoustic fluid are shown in Figs. 7 through 14. It is seen that the non-radiating mode, which for the single shell is to be found at a frequency $\Omega = \Omega_1$, no longer exists for the double shell construction. The multi peaked response which occurs in all of these figures indicates that the separating fluid has the effect of making the concentric shell structure into an infinite number of degrees of freedom system (six, if one were to assume an incompressible separating fluid) for each value of n and l. While for the single shell, in general, three in vacuo natural frequencies are present, for the concentric shell structure an infinite number of frequencies are obtained. Thus in certain regions the pressure in the outer fluid at the interface is somewhat greater for the double shell than for the single shell. However, the infinite response at Ω_1 no longer is present for the double shell, and the pressure ratio drops to about 0.7 for n = 0, L = 4 (Fig. 9) and to the relatively large value of 22.6 when n = 2, L = 4 (Fig. 13). The low pressure occuring in the former case is a consequence of the noncoincidence of the frequency of the non-radiating free vibrational mode and that of the lowest in vacuo mode of the concentric shell.

Finally it has been demonstrated that the introduction of structural damping results in improved transmission loss properties for the double shell.

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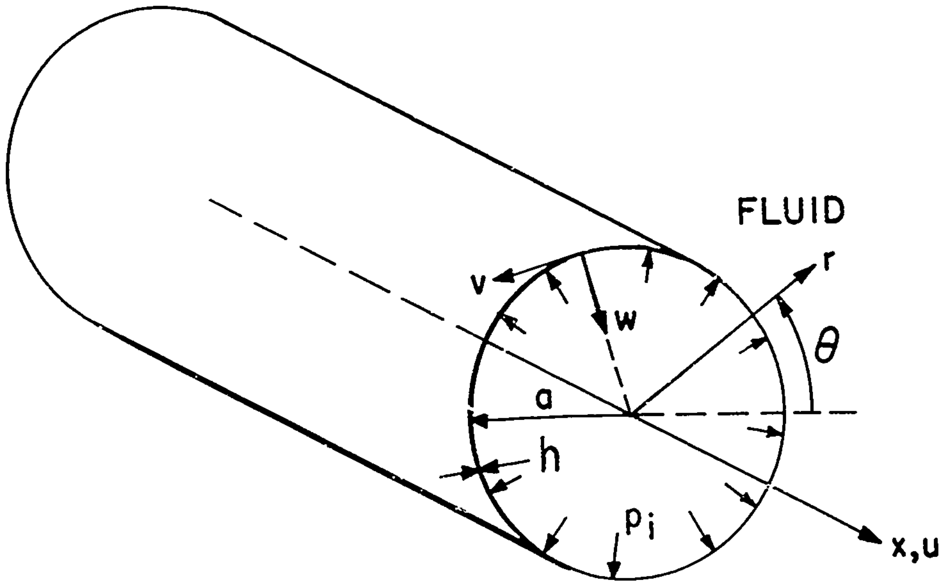

$$p_i = P_i \cos n\theta \cos \frac{\pi x}{L} e^{i\Omega\tau}$$

FIG. 1: GEOMETRY OF THE SINGLE SHELL

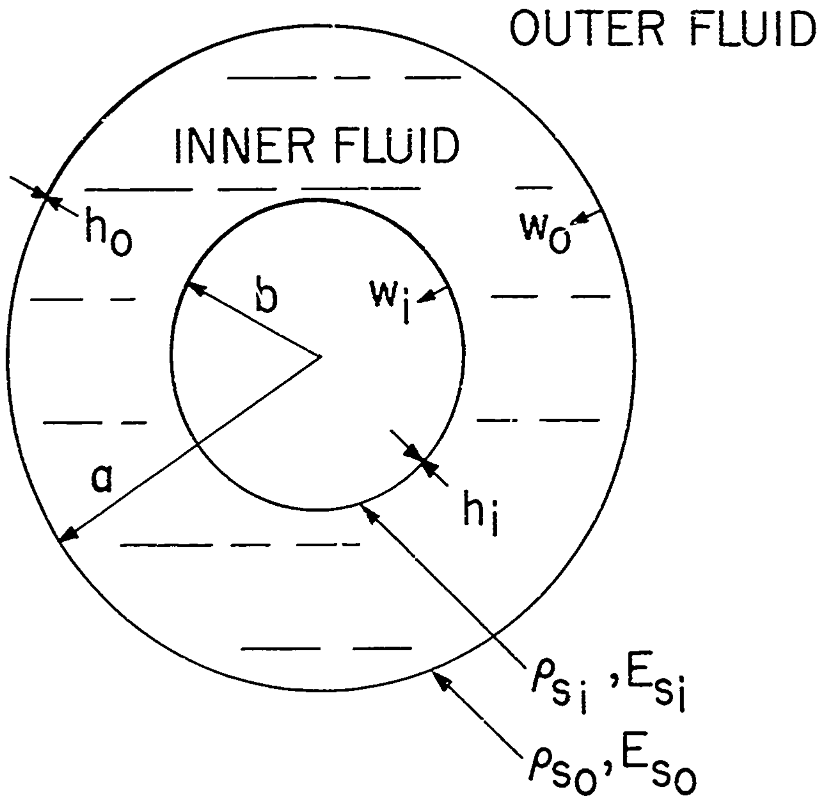

FIG. 2 : CONCENTRIC SHELLS SEPARATED BY AN ACOUSTIC MEDIUM

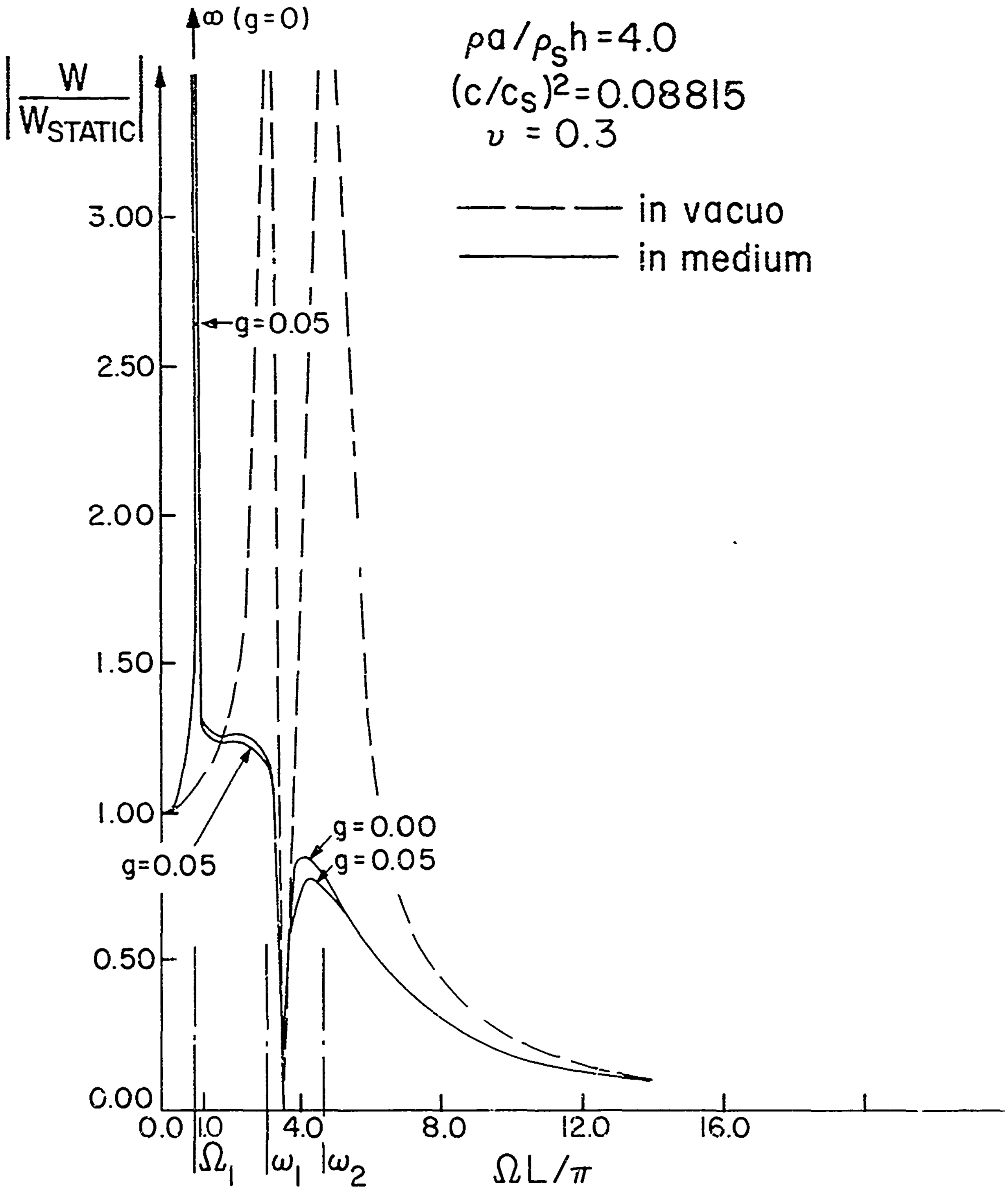

FIG. 3 SINGLE SHELL: DISPLACEMENT VERSUS FREQUENCY RATIO ($n=0, L=4$)

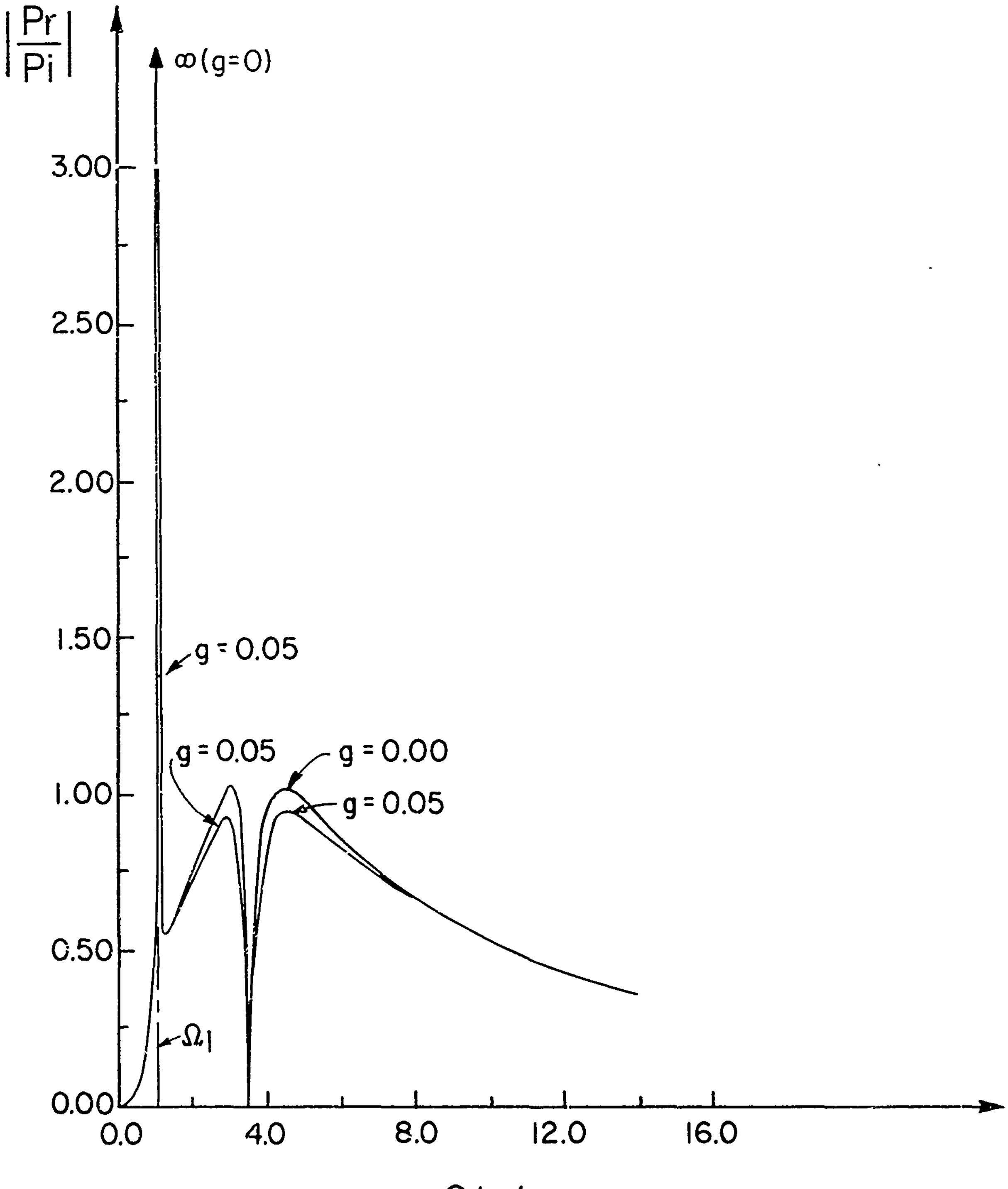

FIG. 4: SINGLE SHELL: PRESSURE (at r = 1.0) VERSUS FREQUENCY RATIO (n = 0, L = 4)

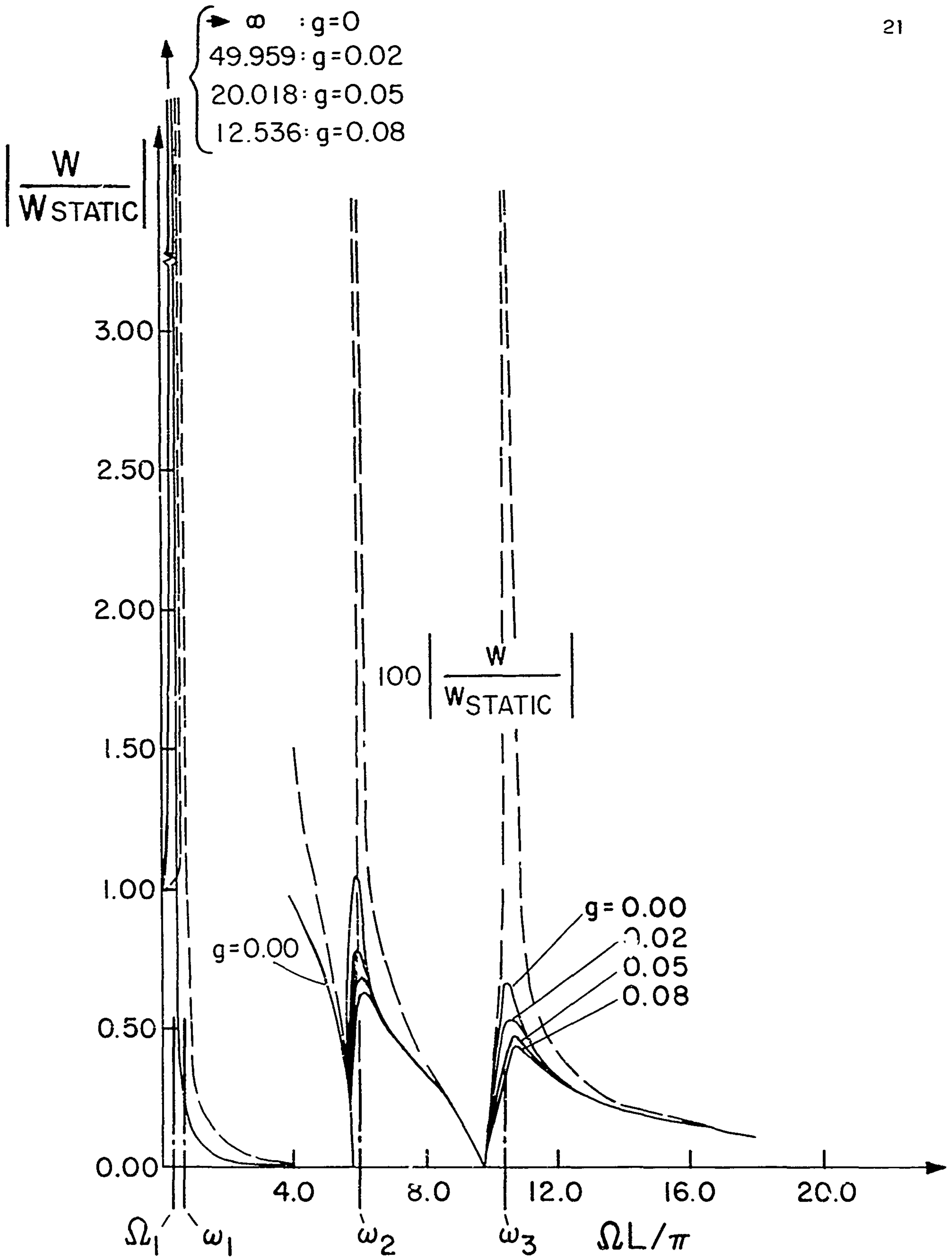

FIG.5 SINGLE SHELL : DISPLACEMENT VERSUS FREQUENY RATIO (n=2, L=4)

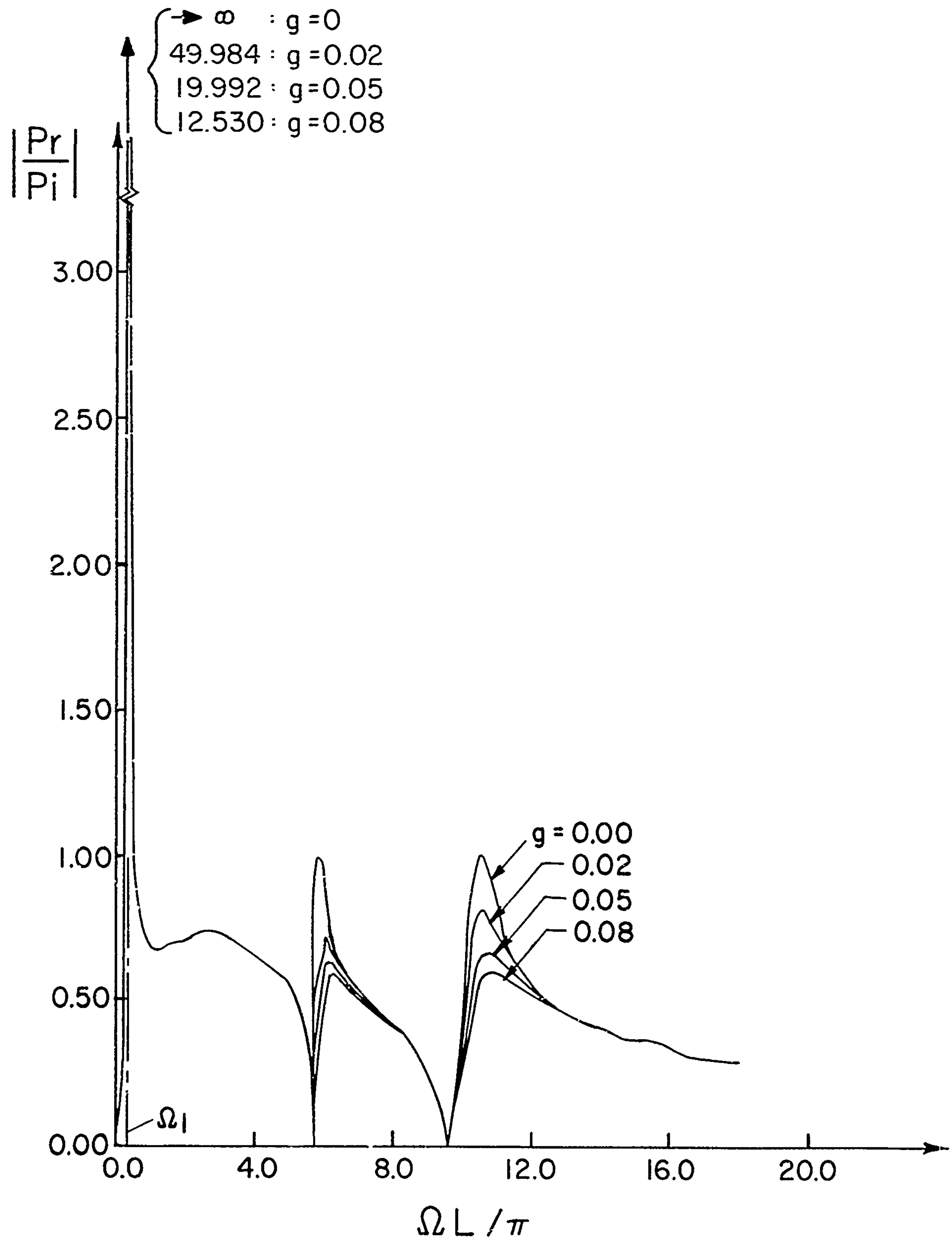

FIG. 6 SINGLE SHELL: PRESSURE (at r = 1.0) VERSUS FREQUENCY RATIO (n=2, L=4)

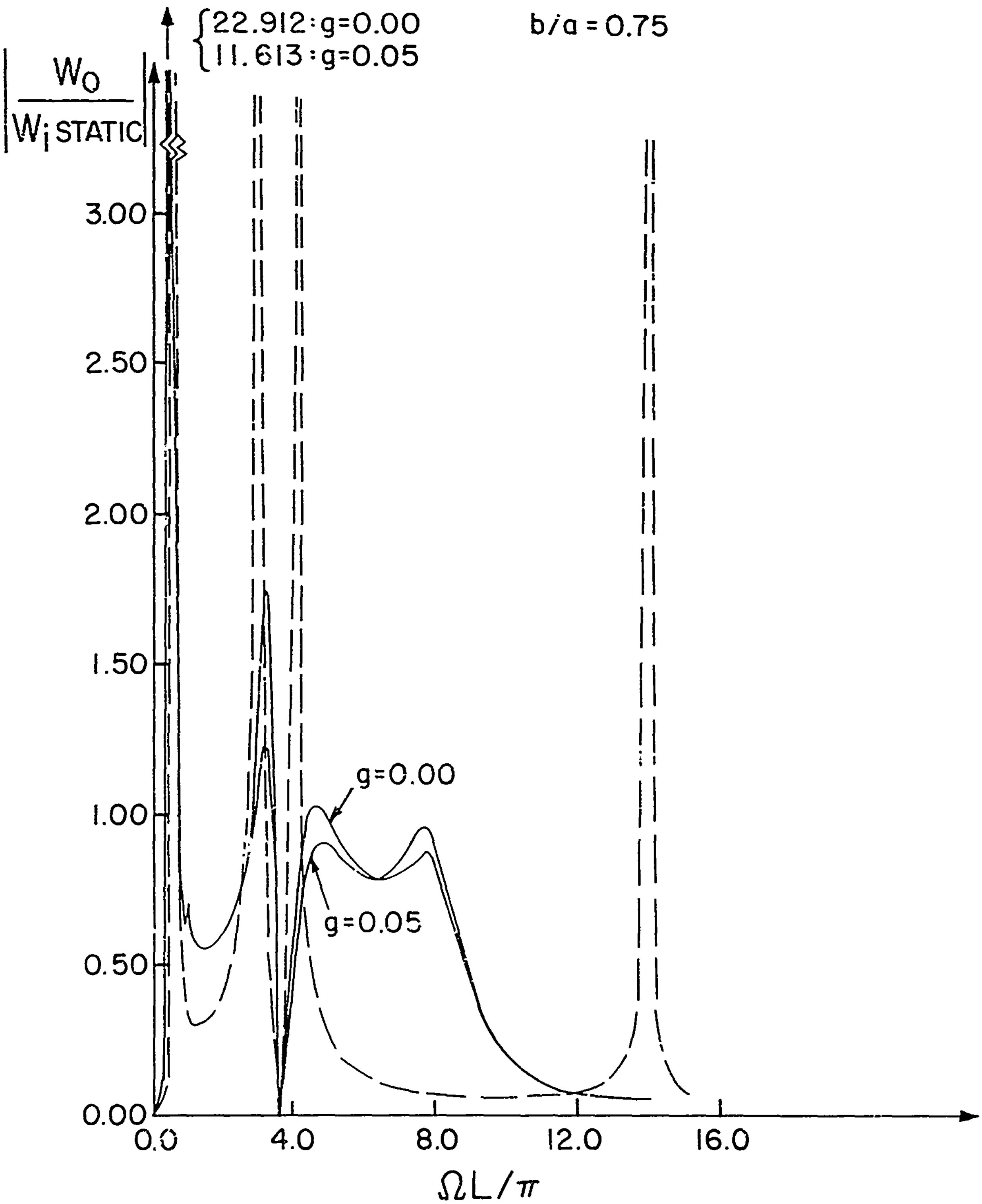

FIG.7: CONCENTRIC SHELLS: DISPLACEMENT OF OUTER SHELL VERSUS FREQUENCY RATIO (n = 0, L = 4)

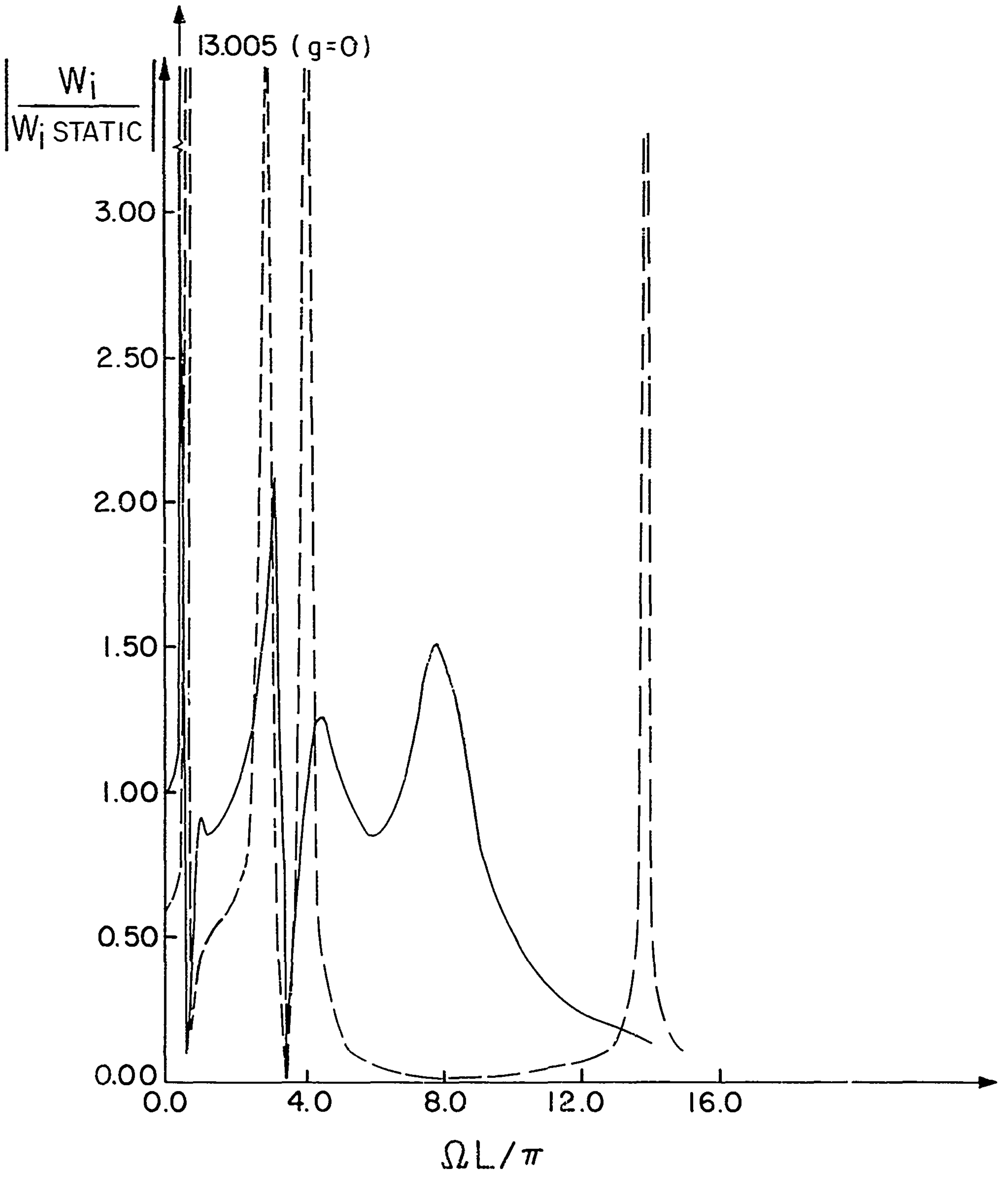

FIG. 8: CONCENTRIC SHELLS : DISPLACEMENT OF INNER SHELL VERSUS FREQUENCY RATIO (n=o, L=4)

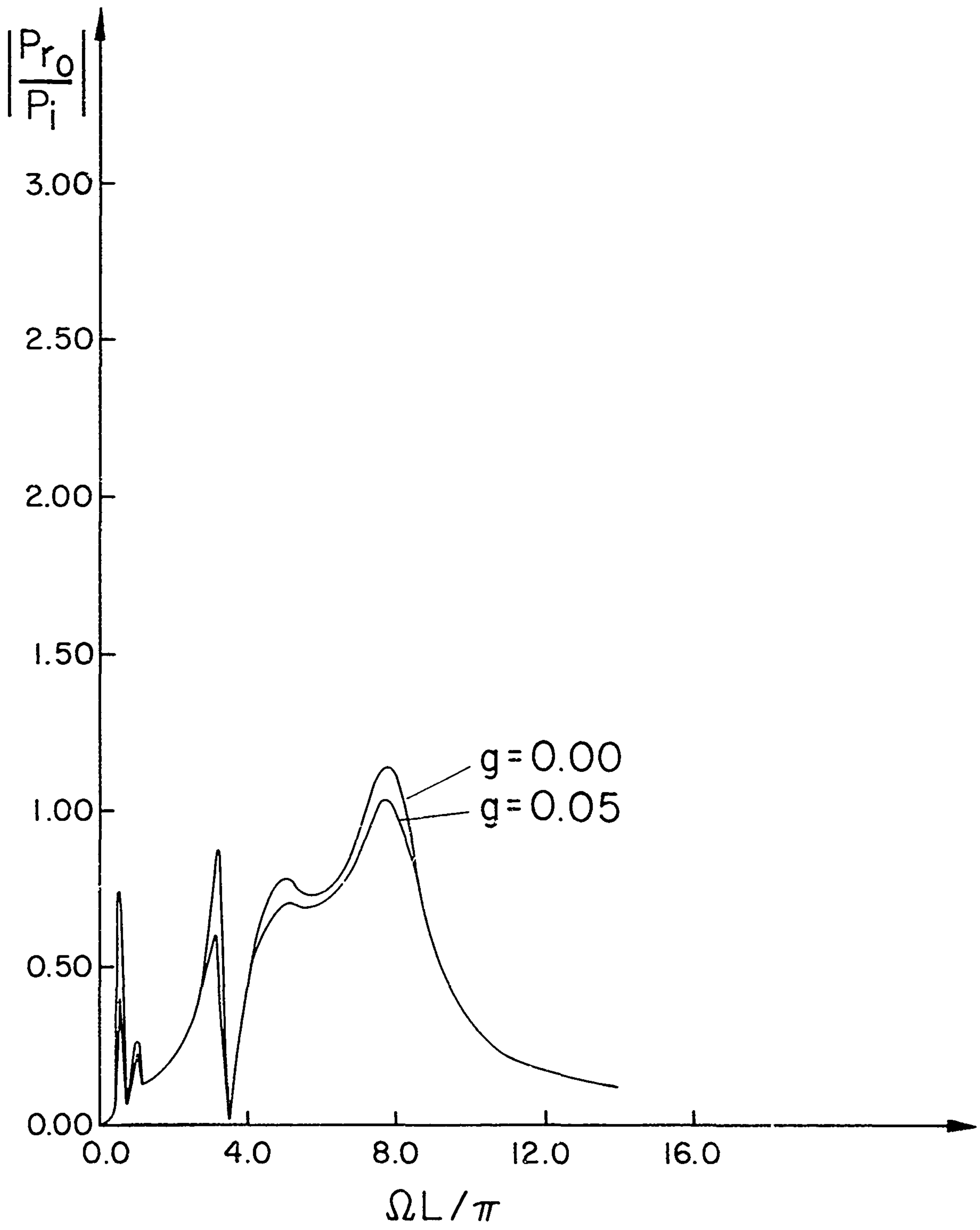

FIG. 9: CONCENTRIC SHELLS: PRESSURE (at r = 1.0) IN OUTER FLUID VERSUS FREQUENCY RATIO (n = 0, L = 4)

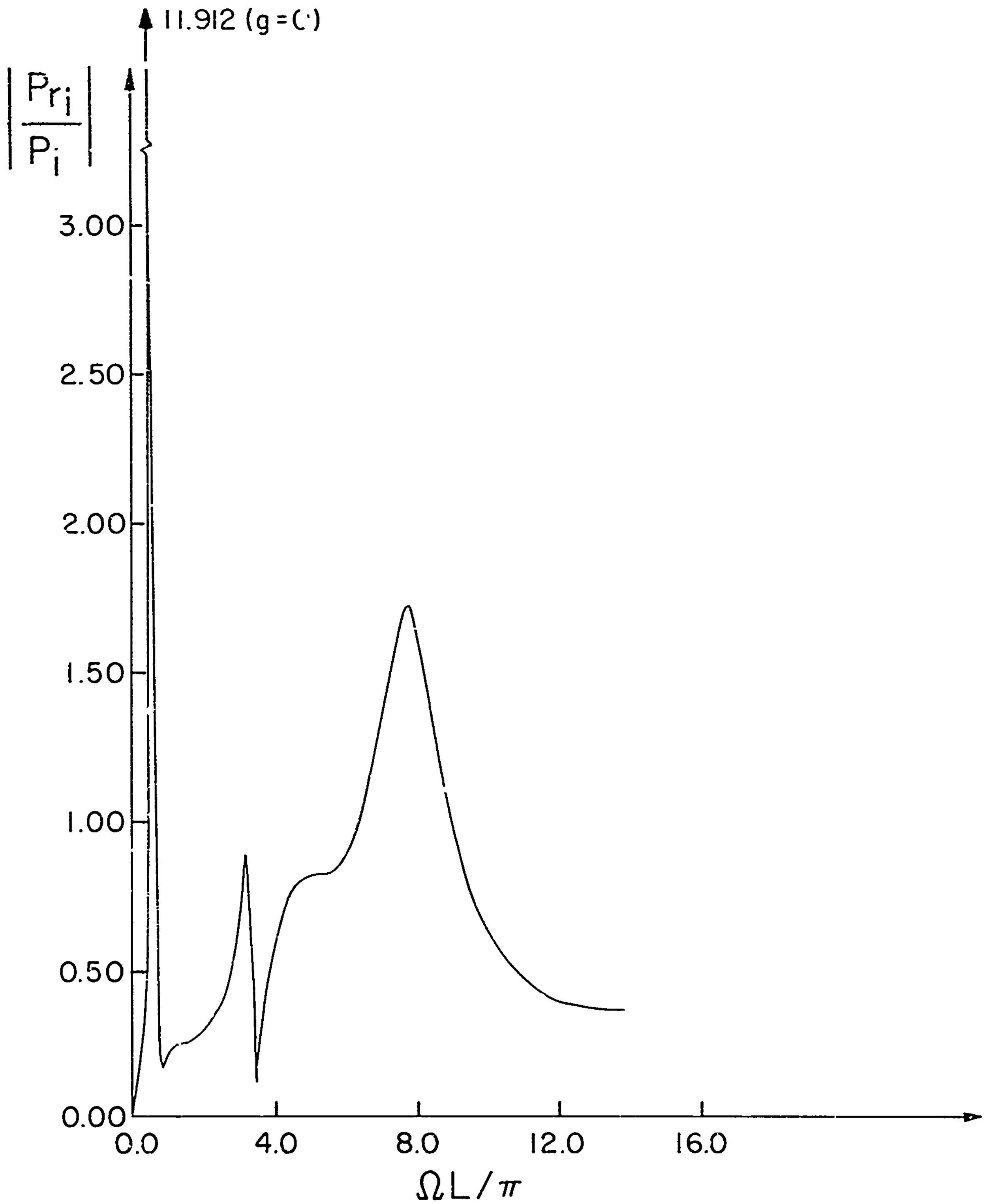

FIG. 10 CONCENTRIC SHELL: PRESSURE (at r=1.0) IN INNER FLUID VERSUS FREQUENCY RATIO (n=0, L=4)

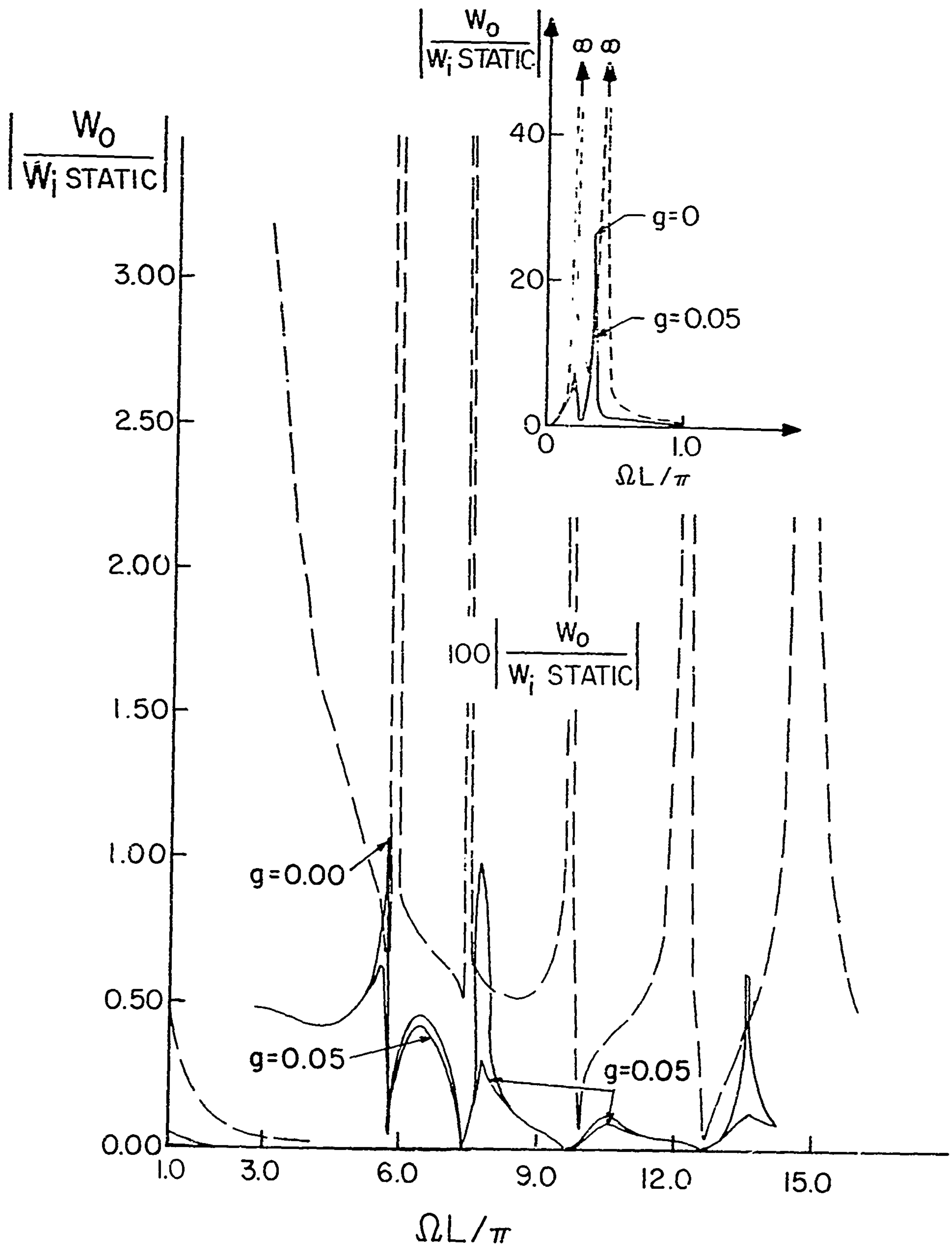

FIG. 11: CONCENTRIC SHELLS : DISPLACEMENT OF OUTER SHELL VERSUS FREQUENCY RATIO (n=2, L=4)

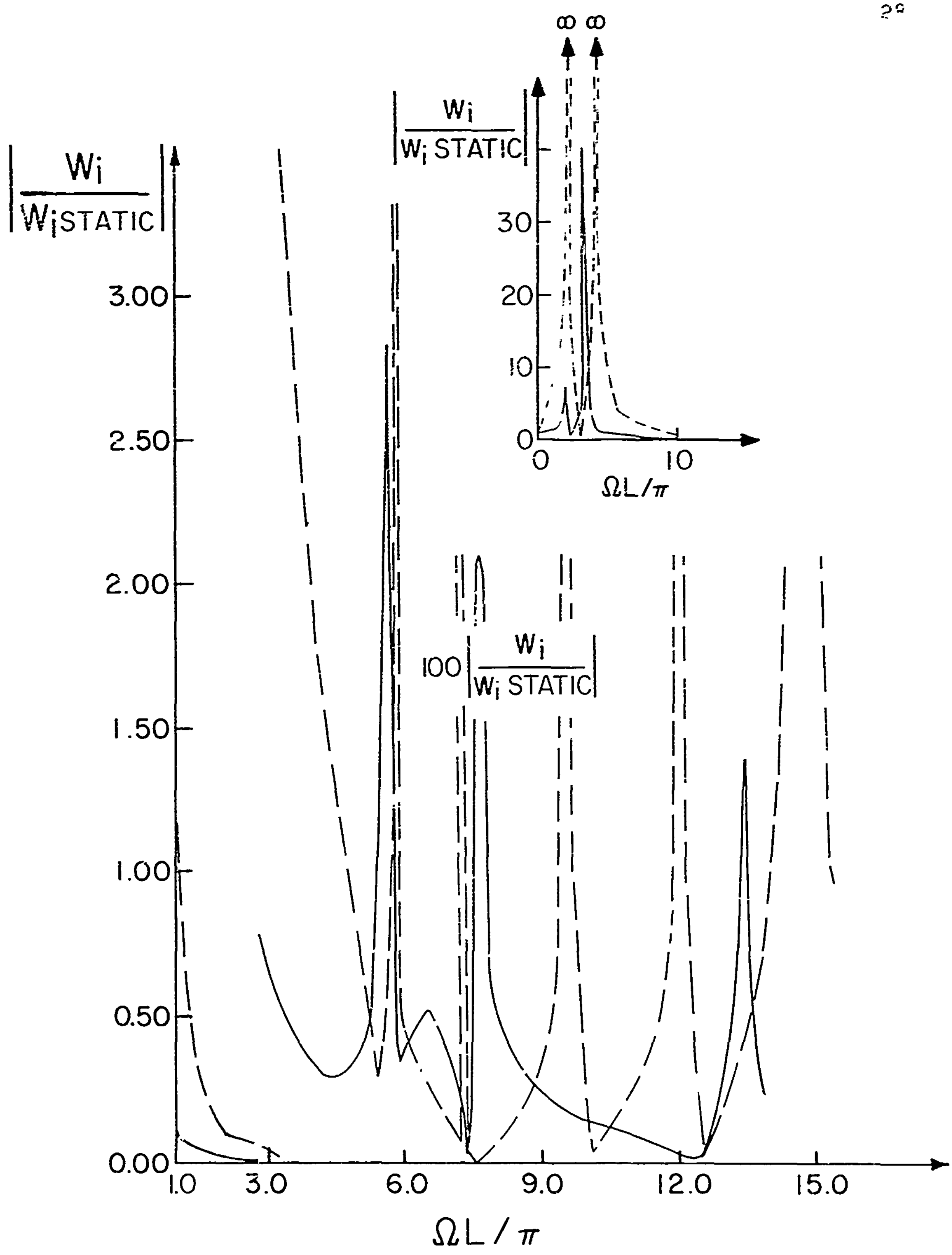

FIG. 12: CONCENTRIC SHELLS: DISPLACEMENT OF INNER SHELL VERSUS FREQUENCY RATIO (n = 2, L = 4)

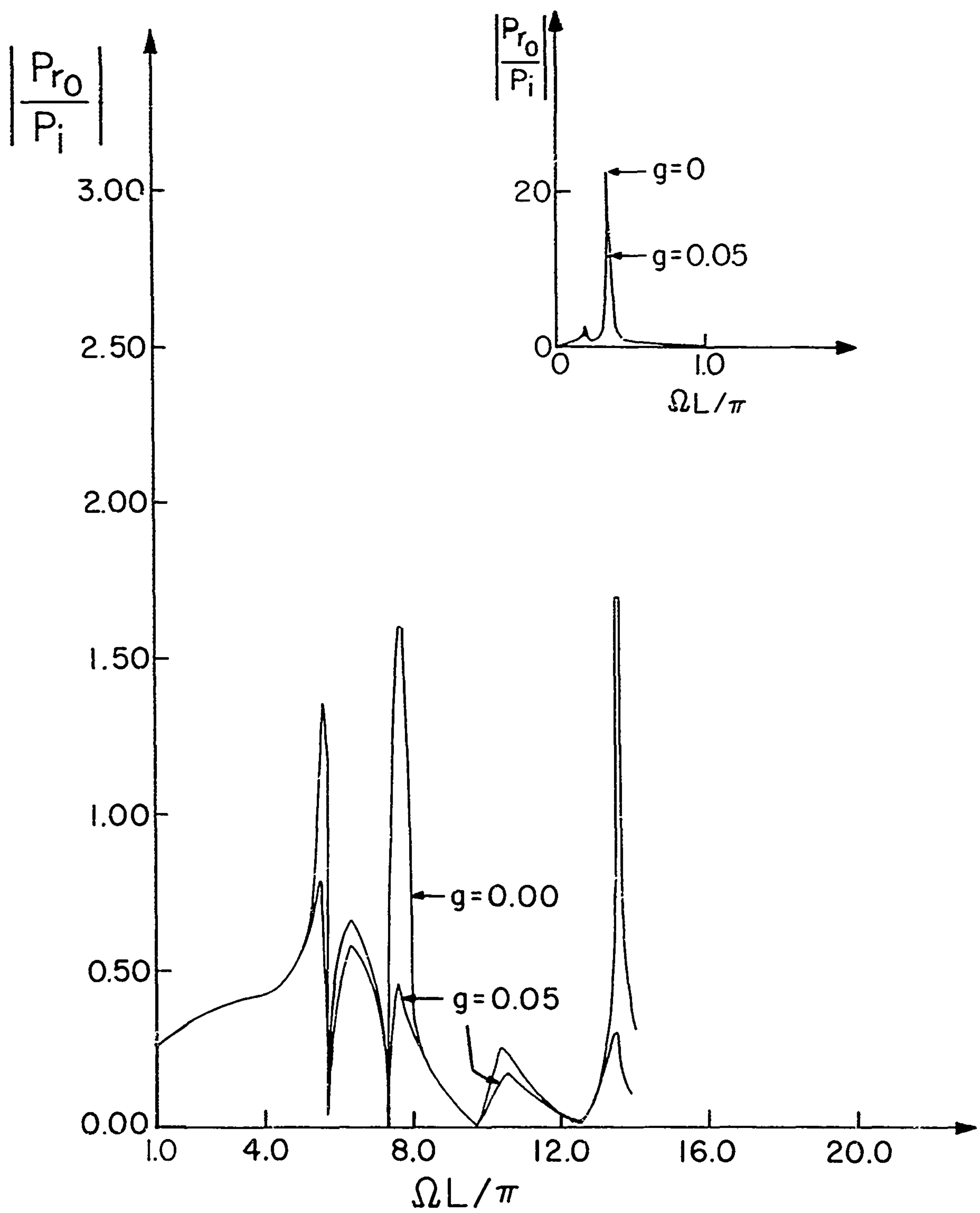

FIG. 13 CONCENTRIC SHELLS : PRESSURE (at r=1.0) IN OUTER FLUID VERSUS FREQUENCY RATIO (n=2, L=4)

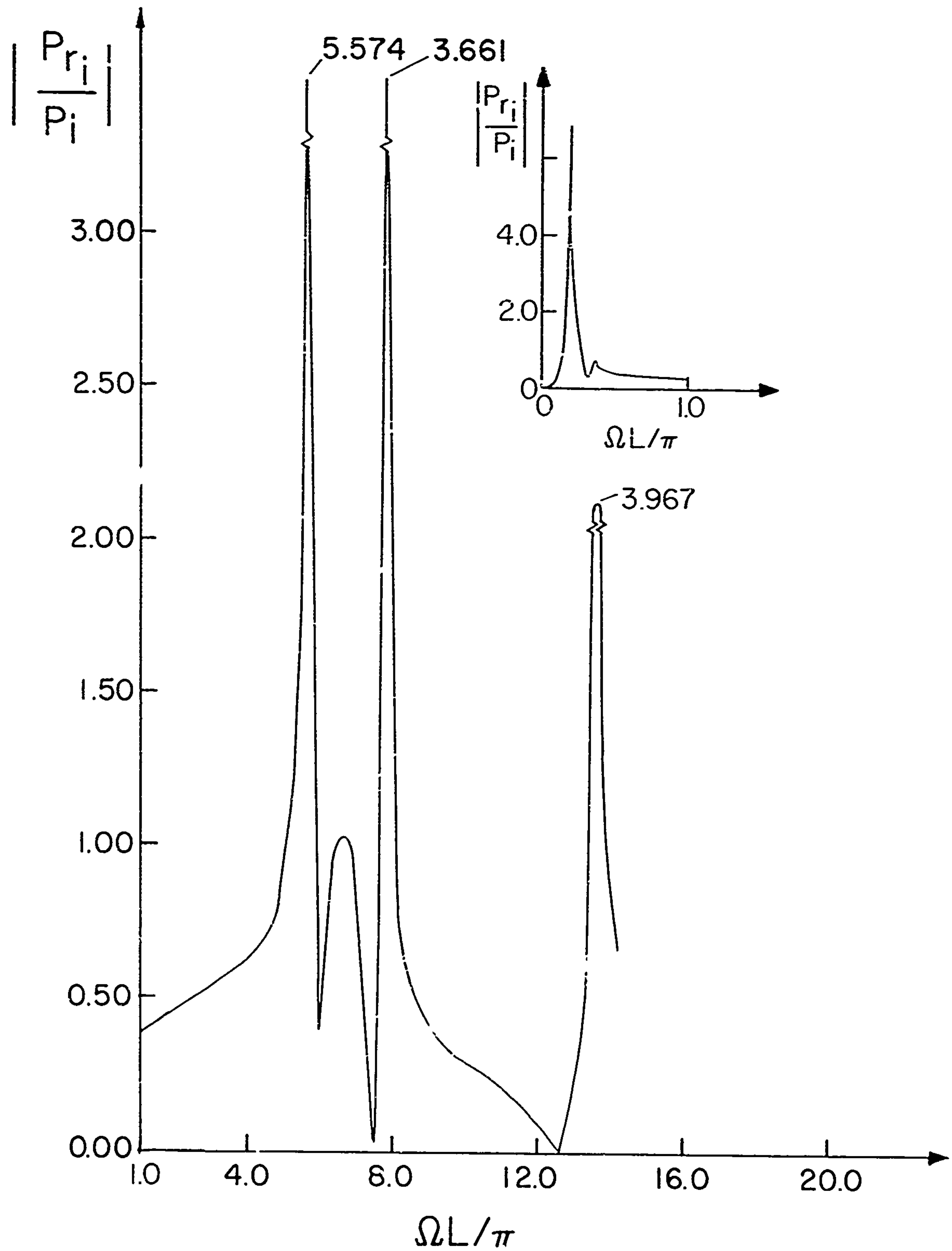

FIG. 14: CONCENTRIC SHELLS: PRESSURE (at r=1.0) IN INNER FLUID VERSUS FREQUENCY RATIO (n=2, L=4)

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